Check-Up

32

Sharing a quantity in a given ratio

a) A primary school headteacher has additional funding of £3640 to be distributed between the infant and junior sections of the school in the ratio 3:4. How much goes to each section? (No calculator needed.)

b) The governing body of a secondary school is concerned about the low ratio of young teachers (under 30) to older teachers and determines to improve this ratio from its current 1:4.5 to 1:3.5. How is the staff of 66 teachers currently distributed between young and older teachers? How will it be distributed if the target is achieved?

c) In the year 2000, the pupils who achieved GCSE passes at grade C or above in all four of English, mathematics, science and a modern foreign language were distributed between boys and girls in a ratio of about 5:7 in favour of girls. About what percentage were boys and what percentage girls?

Answers to check-up 32

a) £1560 to the infants, £2080 to the juniors.

b) Currently: 12 young, 54 older. Target: 14.67 young, 51.33 older.

c) 41.7% boys, 58.3% girls.

Discussion and explanation of check-up 32

In the examples in this check-up, there is a number or quantity that has to be split into two portions, in such a way that a particular ratio between the two parts is achieved.

In example (a), the £3640 has to be shared in the ratio 3:4. This means that if we divide it up into 7 equal parts, we have to give 3 of these parts to the infants and 4 of them to the juniors. Now, £3640 ÷ 7 = £520; so that's £520   3 = £1560 for the infants and £520   4 = £2080 for the juniors. Finally, we just check that the sums distributed do actually add up to the total we started with: £1560 + £2080 = £3640.

This example demonstrates the procedure for sharing a quantity in a given ratio. In general, to share Z in the ratio *a:b*, (i) divide Z by the sum of *a* and *b*; (ii) multiply the result separately by *a* and *b* to get the two shares; (iii) check that the results do add to Z.

Before the first step, it may also be helpful to simplify the ratio. So, in example (b), to share 66 teachers between young and older in the ratio 1:4.5, I would start by rewriting the ratio as 2:9, to get rid of the decimals. So we now divide the 66 by 11 (2 + 9), to get 6, and multiply this separately by the 2 and the 9, to get 12 young teachers and 54 older teachers. Check: 12 + 54 = 66.

Then, to deal with the target of 1:3.5, I would again simplify the ratio, to 2:7. Next we have to divide the 66 by 9 (2 + 7) and multiply separately by 2 and 7. Using a calculator for this, I get 14.666667 and 51.333333. You may suggest, reasonably, that these must be rounded to whole numbers (15 young and 51 older to pass the target). However, schools do employ a number of part-time teachers and teachers on job-shares, so calculations about staffing often involve fractions of teachers. So I have chosen to round the answers to 14.67 and 51.33. Check: 14.67 + 51.33 = 66.

Because 'percent' means 'out of 100', example (c) comes down to sharing 100% between boys and girls in the ratio 5:7. That's $100 \div 12$ $5 = 41.7\%$ boys, and $100 \div 12$ $7 = 58.3\%$ girls. Another way of looking at this is to say that $\frac{5}{12}$ of the pupils are boys and $\frac{7}{12}$ are girls, and then change these fractions to their percentage equivalents.

Summary of key ideas

◆   To share a quantity or number Z into two portions in the ratio *a:b* (e.g. share £24 in the ratio 3:5):

– divide Z by the sum of *a* and *b* (£24 ÷ 8 = £3)

– multiply the result separately by *a* and *b* to get the two shares (£3   3 = £9, £3   5 = £15)

– check that the two shares add to Z (£9 + £15 = £24).

◆   If a quantity or number is shared in the ratio *a:b* then the fraction in the first portion is $\frac{a}{(a+b)}$ and the fraction in the second portion is $\frac{b}{(a+b)}$ (for example, sharing in the ratio 3:5 results in $\frac{3}{8}$ in the first portion and $\frac{5}{8}$ in the second).

Further practice

32.1 The ideas in this check-up can also be applied to sharing quantities into three or more parts. For example, share a grant of £4800 between the nursery, infant and junior sections of a primary school in the ratio 1:2:5.

32.2 The entrants for GCSE music in the year 2000 were split between girls and boys in the ratio 4:3. What fraction of the entrants were boys and what fraction girls? Express the proportions of girls and boys as percentages of the total number of entrants. There were approximately 41 000 entrants. So, to the nearest hundred, about how many were girls and how many boys?